

# Creation of Fundamental Strings by Crossing D-branes

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## Abstract

We study the force balance between orthogonally positioned  $p$ -brane and  $(8-p)$ -brane. The force due to graviton and dilaton exchange is repulsive in this case. We identify the attractive force that balances this repulsion as due to one-half of a fundamental string stretched between the branes. As the  $p$ -brane passes through the  $(8-p)$ -brane, the connecting string changes direction, which may be interpreted as creation of one fundamental string. We show this directly from the structure of the Chern-Simons terms in the D-brane effective actions. We also discuss the effect of string creation on the 0-brane quantum mechanics in the type I' theory. The creation of a fundamental string is related by U-duality to the creation of a 3-brane discussed by Hanany and Witten. Both processes have a common origin in M-theory: as two M5-branes with one common direction cross, a M2-brane stretched between them is created.

# 1 Introduction

The Dirichlet branes [1, 2] are a remarkable window into non-perturbative string theory. The D-branes are BPS saturated objects which preserve 16 supersymmetries. This implies that when two Dirichlet  $p$ -branes are placed parallel to each other, the force between them vanishes. A string theoretic calculation of this force involves the cylinder diagram, with the ends of the cylinder attached to different D-branes. In the open string channel there are contributions from the NS, R, and NS  $(-1)^F$  sectors, which cancel due to the abstruse identity for theta-functions [2]. Physically, this means that the attraction due to NS-NS closed strings, the graviton and the dilaton, is canceled by the repulsion of the like R-R charges.

It is interesting to study a more general situation where a  $p'$ -brane is placed parallel to a  $p$ -brane with  $p' < p$ . This configuration preserves 8 of the supersymmetries if  $p - p' = 4$  or 8. For  $p - p' = 4$  only the NS and the R open string sectors contribute to the cylinder amplitude, and they cancel identically. Thus, there is no force due to the R-R exchange, while the graviton and the dilaton forces cancel identically.

A more complicated situation, which is the main subject of this paper, arises for  $p - p' = 8$ . One example of this is the 0-brane near the 8-brane, which is important for understanding the heterotic theory [3]. Now the contribution of the NS and R open string sectors to the cylinder amplitude is [4]

$$A_{NS-NS} = \frac{1}{2} \int_0^\infty \frac{dt}{t} (8\pi^2 \alpha' t)^{-1/2} e^{-\frac{tY^2}{2\pi\alpha'}} f_4^{-8}(q) \left( -f_2^8(q) + f_3^8(q) \right) = \frac{1}{2} \int_0^\infty \frac{dt}{t} (8\pi^2 \alpha' t)^{-1/2} e^{-\frac{tY^2}{2\pi\alpha'}} \quad (1)$$

where  $q = e^{-\pi t}$  and  $Y$  is the transverse position of the 0-brane relative to the 8-brane. Thus, we find a constant repulsive force due to the NS-NS closed strings,

$$- \frac{\partial A_{NS-NS}(Y)}{\partial Y} = \frac{1}{4\pi\alpha'} \frac{Y}{|Y|} . \quad (2)$$

As pointed out by Lifschytz [4], this is canceled by a contribution of the R  $(-1)^F$  open string sector, which implies that there is attraction due to R-R exchange. How can we understand this attraction physically? We will try to clarify this issue.

A peculiar feature of this force is that it jumps by  $\pm \frac{1}{2\pi\alpha'}$  every time the 0-brane crosses the 8-brane. Our main point is that this jump is due to **creation** of a fundamental string stretched between the 0-brane and the 8-brane. This phenomenon is similar, and in fact U-dual, to the creation of a 3-brane discovered by Hanany and Witten [5]. Since the number of stretched fundamental strings jumps by  $\pm 1$  upon each crossing, we may regard the ground state of the 0-8 system as containing  $\pm \frac{1}{2}$  of a fundamental string (the sign refers to the direction of the string). When the 0-brane is to the left of the 8-brane, we have, say,  $-\frac{1}{2}$  of a fundamental string. Upon crossing, this turns into  $+\frac{1}{2}$  because the string changes direction. Indeed, the attractive force equal to  $\frac{1}{2}$  of the fundamental string tension is what is necessary to cancel the repulsion due to the graviton and the dilaton, (2). This is how the no-force condition required by supersymmetry is maintained in the 0-8 system.

## 2 U-duality and creation of a fundamental string

In this section we show that the creation of a stretched string by a 0-brane crossing an 8-brane is related by U-duality to creation of a stretched 3-brane by a R-R 5-brane crossing a NS-NS 5-brane. Hanany and Witten showed that, when an R-R charged 5-brane positioned in the  $(1 - 2 - 6 - 7 - 8)$  directions crosses a NS-NS charged 5-brane positioned in the  $(1 - 2 - 3 - 4 - 5)$  directions, a single  $(1 - 2 - 9)$  3-brane stretched between the 5-branes is created [5].

Applying T-duality along directions 1 and 2 we find that, when a  $(6 - 7 - 8)$  3-brane crosses a  $(1 - 2 - 3 - 4 - 5)$  NS-NS 5-brane, then a D-string stretched between them along the 9th direction is created. From the S-duality of the type IIB theory it now follows that, when a  $(6 - 7 - 8)$  3-brane crosses a  $(1 - 2 - 3 - 4 - 5)$  R-R 5-brane, then a fundamental string stretched between them along the 9th direction is created. This is the kind of process that is of primary interest to us, because it involves two D-branes with 8 ND coordinates. There are a number of other such processes related to this by T-duality. For example, after T-dualizing along directions 6, 7 and 8, we find that a 0-brane crossing an 8-brane creates a stretched fundamental string.

It is interesting that both the Hanany-Witten process and the fundamental string creation originate from the same phenomenon in M-theory: creation of a 2-brane by crossing 5-branes. Indeed, when a  $(2 - 3 - 4 - 5 - 10)$  5-brane crosses a  $(6 - 7 - 8 - 9 - 10)$  5-brane, a  $(1 - 10)$  2-brane stretched between the 5-branes is created. Reducing to the type IIA theory along direction 5, we find that a 4-brane crossing a 5-brane creates a 2-brane. This is T-dual to the 3-brane creation discussed in [5]. We may, however, choose to reduce to the type IIA theory along direction 10, which is common to all the branes. Then we find that a  $(2 - 3 - 4 - 5)$  4-brane crossing a  $(6 - 7 - 8 - 9)$  4-brane creates a fundamental string stretched along direction 1. This confirms that two crossing D-branes, positioned in such a way that there are 8 ND coordinates, create a stretched fundamental string.

## 3 Effective action arguments

In this section we give a direct argument for the creation of fundamental strings, independent of the result in [5]. We will rely on the well-known structure of the D-brane effective actions. For concreteness, we will refer to the 0-8 system, but analogous arguments apply to all cases related to this by T-duality.

The term in the 8-brane world volume action which is crucial for our purposes is [2]

$$\mu_{(8)} \frac{1}{2 \cdot 7!} \int d^9 \sigma \varepsilon_{\nu_0 \dots \nu_8} C_{(7)}^{\nu_0 \dots \nu_6} F^{\nu_7 \nu_8} , \quad (3)$$

where  $C_{(7)}$  is an R-R potential, and  $F = dA$  is the world volume gauge field strength. The D-brane charge densities were determined in [2] to be

$$\mu_{(p)} = \sqrt{2\pi} (2\pi\sqrt{\alpha'})^{3-p} .$$

Integrating (3) by parts, we get

$$\mu_{(8)} \frac{1}{8!} \int d^9 \sigma \varepsilon_{\nu_0 \dots \nu_8} F_{(8)}^{\nu_0 \dots \nu_7} A^{\nu_8} = \mu_{(8)} \int d^9 \sigma F_{(2)}^{\mu 9} A_\mu , \quad (4)$$

where

$$F_{(8)} = dC_{(7)} , \quad F_{(2)} = {}^* F_{(8)} ,$$

and 9 is the direction normal to the 8-brane.

In the presence of a stationary 0-brane, there is a radial electric field,

$$F_{(2)}^{0r} = \frac{\mu_{(0)}}{r^8 \Omega_8} ,$$

where  $\Omega_8$  is the volume of a unit 8-sphere. Eq. (4) shows that the normal component of the electric field,  $F_{(2)}^{09}$ , plays the role of the charge density in the world volume gauge theory. The total charge on the 8-brane is

$$\mu_{(8)} \int d^8 \sigma F_{(2)}^{09} = \frac{1}{2} \mu_{(8)} \mu_{(0)} = \frac{1}{4\pi\alpha'} .$$

Let us recall that an endpoint of a fundamental string manifests itself in the world volume gauge theory as an electric charge of magnitude  $\pm \frac{1}{2\pi\alpha'}$ . We conclude that the 0-brane and the 8-brane are connected by **one half** of a fundamental string. This provides the attraction that cancels the repulsion from the graviton-dilaton exchange.

As the 0-brane crosses the 8-brane, the net electric charge on the 8-brane jumps from  $\frac{1}{4\pi\alpha'}$  to  $-\frac{1}{4\pi\alpha'}$ . This clearly shows that an endpoint of a fundamental string is created on the 8-brane. Similar considerations in the 0-brane action are expected to show that the other end of the string is attached to the 0-brane. The term in the 0-brane action responsible for this effect appears to be

$$\mu_{(0)} \int d\tau F A_0 , \quad (5)$$

where  $F = {}^* F_{(10)}$  is the zero-form field strength dual to the 10-form emitted by the 8-brane. Thus,  $\mu_{(0)} F$  is the ‘source’ for  $A_0$ . We believe that this shows that the fundamental string indeed ends on the 0-brane. Correctness of this argument may be checked through T-duality. For instance, if we T-dualize the 0-8 system to a pair of orthogonal 4-branes, then (5) goes into the following term of the 4-brane action,

$$\frac{\mu_{(4)}}{4!} \int d^5 \sigma \varepsilon_{\nu_0 \dots \nu_4} F_{(4)}^{\nu_0 \nu_1 \nu_2 \nu_3} A^{\nu_4} . \quad (6)$$

The jump in the total charge on a 4-brane as it is crossed by the other 4-brane is

$$\mu_{(4)}^2 = \frac{1}{2\pi\alpha'} ,$$

which is precisely the tension of one fundamental string.

In the previous section we showed that the string creation follows by dimensional reduction from membrane creation in M-theory. Let us make a direct argument for the latter.

Consider the effective action for a  $(1-2-3-4-5)$  5-brane in the presence of a  $(1-6-7-8-9)$  5-brane. This action contains a Chern-Simons term

$$q_{(5)} \frac{1}{(3!)^2} \int d^6 \sigma \varepsilon_{\nu_0 \dots \nu_5} C^{\nu_0 \nu_1 \nu_2} H^{\nu_3 \nu_4 \nu_5} , \quad (7)$$

where  $H = dB$  is the world volume field strength. The 2-brane and 5-brane charge densities and tensions were normalized in [6, 7],

$$q_{(2)} = \sqrt{2} \kappa T_{(2)} = \sqrt{2} (2\kappa \pi^2)^{1/3} , \quad (8)$$

$$q_{(5)} = \sqrt{2} \kappa T_{(5)} = \sqrt{2} \left(\frac{\pi}{2\kappa}\right)^{1/3} . \quad (9)$$

Integrating (7) by parts, we find

$$q_{(5)} \frac{1}{2 \cdot 4!} \int d^6 \sigma \varepsilon_{\nu_0 \dots \nu_5} F^{\nu_0 \nu_1 \nu_2 \nu_3} B^{\nu_4 \nu_5} . \quad (10)$$

This shows that  $F_{2345}$  acts as a source for  $B_{01}$ . Thus,  $F_{2345}$  is proportional to the density of strings on the world volume which point along direction 1. Such a string is the boundary of a  $(1-10)$  2-brane stretched between the 5-branes. Evaluating the flux through the  $(1-2-3-4-5)$  5-brane due to the  $(1-6-7-8-9)$  5-brane,

$$\int d^5 \sigma F_{2345} ,$$

we find that the net charge that couples to  $B_{01}$  is

$$\frac{q_{(5)}^2}{2} = \frac{T_{(2)}}{2} . \quad (11)$$

Thus, **one half** of a 2-brane is stretched between the 5-branes. As the 5-branes pass through each other, one 2-brane is created. It is interesting that this process is encoded in the relation (11) between the charge of the 5-brane and the tension of the 2-brane in M-theory.

## 4 String creation in heterotic D-particle quantum mechanics

The phenomena we have discussed can also be understood from the point of view of the quantum mechanical system introduced in [3] and elaborated on in [8, 9, 10, 11, 12]. We will study a system with gauge group  $SO(2)$ , i.e. a D-particle interacting with its mirror image in the background of 8 D8-branes (+ mirror images) and an 8-orientifold. The Hamiltonian is given by (this is valid for  $SO(N)$  generically)

$$\begin{aligned} H = & \text{Tr} \left\{ \lambda_{I'} \left( \frac{1}{2} P_i^2 - \frac{1}{2} E_9^2 \right) + \frac{1}{\lambda_{I'}} \left( \frac{1}{2} [A_9, X_i]^2 - \frac{1}{4} [X_i, X_j]^2 \right) \right. \\ & \left. + \frac{i}{2} \left( -S_a [A_9, S_a] - S_{\dot{a}} [A_9, S_{\dot{a}}] + 2X_i \sigma_{a\dot{a}}^i \{S_a, S_{\dot{a}}\} + \sum_{i=1}^{16} \chi_i^I A_{9IJ} \chi_i^J \right) \right\} , \quad (12) \end{aligned}$$

where  $P_i$  are the conjugate momenta of the  $X_i$  and  $E_9$  the conjugate momentum of  $A_9$ . The bosonic fields  $X_i$  are in the traceless symmetric representation, while  $A_9$  is in the adjoint. For the fermionic fields,  $S_{\tilde{a}}$  is in the traceless symmetric representation,  $S_a$  is in the adjoint while the  $\chi_i$  are in the fundamental.  $A_9$  corresponds to the distance between the D-particle and its mirror. We put  $A_9$  equal to  $r$  and use the Born-Oppenheimer approximation to integrate out the massive fields, i.e. the 16 bosonic modes from  $X_i$ , the 16 real fermionic modes from  $S_{\tilde{a}}$  and the 32 real fermionic modes from  $\chi_i$ . It is found that an effective potential given by

$$V(r) = r \sum_{i=1}^{16} (N_i^B + 1/2) + r \sum_{i=1}^8 (N_i^F - 1/2) + \frac{r}{2} \sum_{i=1}^{16} (N_i^f - 1/2) \quad (13)$$

is generated. The first two terms are the bosonic and fermionic contributions from strings stretching between the D-particle and its mirror image. Since we have only half the number of fermions compared to the situation without an orientifold, [13, 14, 15], the vacuum energies do not cancel unless we add the 8 D8-branes. This will give 16 extra complex fermions (not 8) but their contribution is nevertheless exactly what we need. As seen above there is a relative factor 1/2 in the potential due to the fact that the fermions are only in the fundamental representation. Another way to understand the relative factor is that they correspond to strings going between the D-particle and a D8-brane, rather than between the D-particle and its mirror.

If we separate the D8-branes from the orientifold by turning on Wilson lines the last term in (13) generalizes to

$$\frac{1}{2} \sum_{i=1}^8 \left( |r - m_i| (N_i^{fR} - 1/2) + |r + m_i| (N_i^{fL} - 1/2) \right) \quad (14)$$

Let us start with a supersymmetric, forceless situation with the D-particle to the right of all D8-branes. Then move it through the rightmost D8-brane, say the one with  $i = 8$ . The change of sign provides a net force. This force can however be canceled by the creation of exactly one string, i.e.  $N_8^{fR} = 1$ . This, then, is consistent with the picture that we have developed in the preceding sections.

## Acknowledgments

We are grateful to Pär Stjernberg for useful discussions. I.R.K. thanks the Theoretical Physics group at Uppsala University for its warm hospitality. The work of I.R. Klebanov was supported in part by DOE grant DE-FG02-91ER40671, the NSF Presidential Young Investigator Award PHY-9157482, and the James S. McDonnell Foundation grant No. 91-48.

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